EX 8 :

1) $x \in]2,+\infty[$

$$f(x) = x+1+\frac{4}{(x-2)^2}$$

$$g(x) = x+1$$

(ζ_g) est la droite d'équation : y=x+1

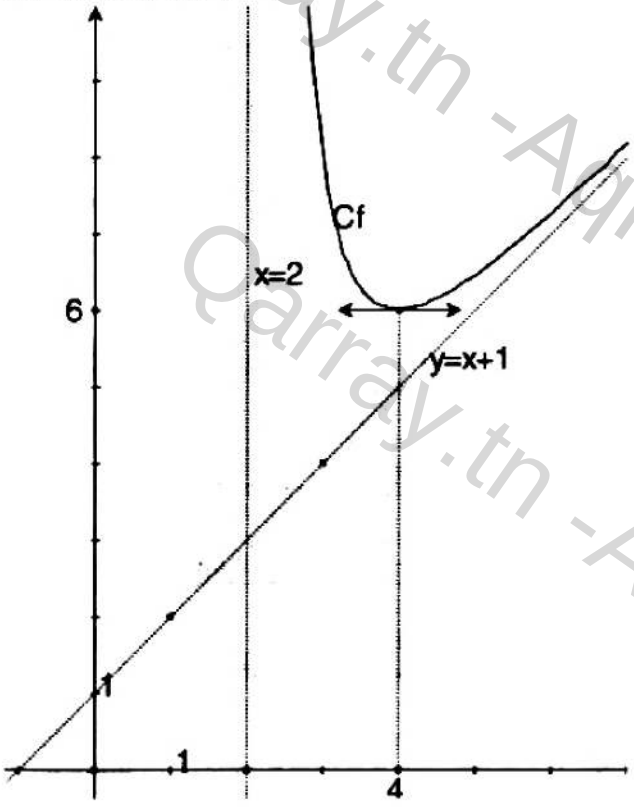
$$f'(x) = 1-\frac{8}{(x-2)^3} = \frac{(x-2)^3-2^3}{(x-2)^3}$$

$$f'(x) = \frac{(x-4)\left[(x-2)^2+2(x-2)+4\right]}{(x-2)^3}$$

x	2		4		$+\infty$
f'(x)	||	-	0	+	
f(x)	|| $+\infty$	↘	6	↗	$+\infty$

$$\lim_{x\to+\infty}\left[f(x)-(x+1)\right] = \lim_{x\to+\infty}\frac{4}{(x-2)^2} = 0$$

(ζ_g) : y=x+1 est une asymptote à (ζ_f) au voisinage de $(+\infty)$



2) $f(x) \geq g(x)$, $\forall x \in]2,+\infty[$

a) si $\lambda \geq 3$

$$A(\lambda) = \int_3^{\lambda}(f(x)-g(x))dx = \int_3^{\lambda}\frac{4}{(x-2)^2}dx$$

si $2<\lambda<3$

$$A(\lambda) = \int_{\lambda}^{3}\frac{4}{(x-2)^2}dx = -\int_3^{\lambda}\frac{4}{(x-2)^2}dx$$

$A(\lambda) \geq 0$

d'où $A(\lambda) = \left|\int_3^{\lambda}\frac{4}{(x-2)^2}dx\right|$

b) $A(\lambda) = \left|\left[\frac{-4}{x-2}\right]_3^{\lambda}\right| = \left|4-\frac{4}{\lambda-2}\right|$

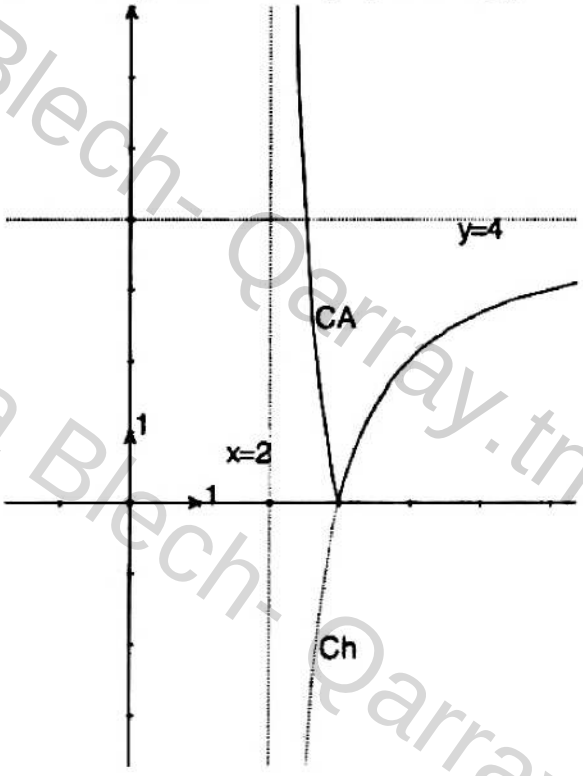
c) soit $h(\lambda) = 4-\frac{4}{\lambda-2}$

$$\begin{cases} A(\lambda) = h(\lambda) & \text{si } \lambda \geq 3 \\ A(\lambda) = -h(\lambda) & \text{si } 2<\lambda<3 \end{cases}$$

$$h'(\lambda) = \frac{4}{(\lambda-2)^2} > 0$$

λ	2		$+\infty$
h'(λ)	||	+	
h(λ)	|| $-\infty$	↗	4

$x = 2$ et y=4 sont les asymptotes à (ζ_h)



Qarray.tn -Aqra Blech- Qarray.tn